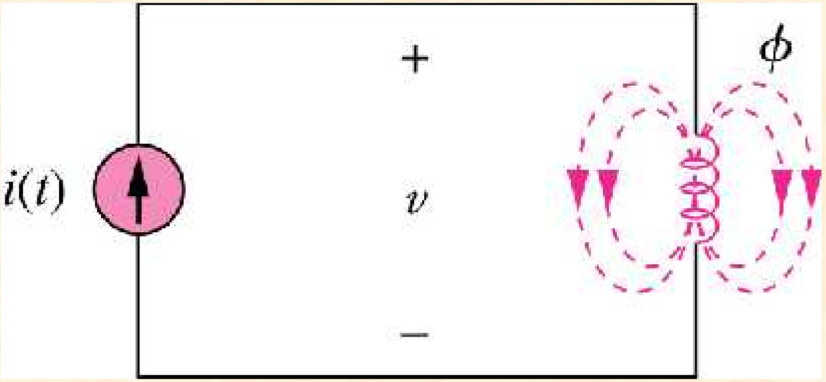
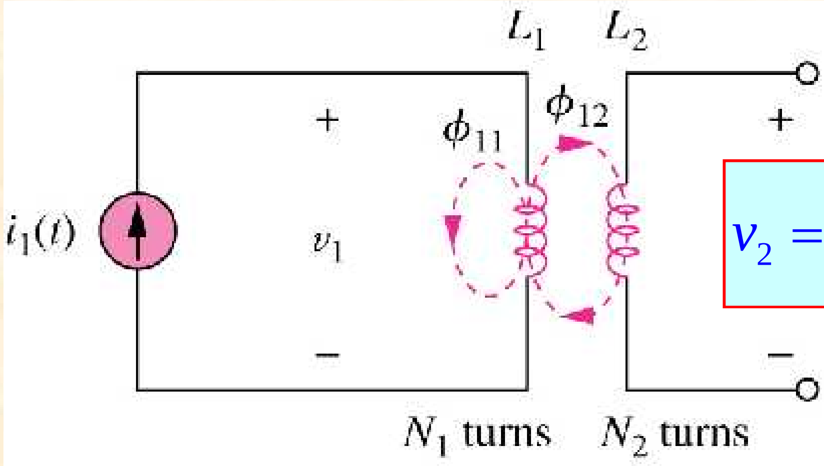
Mutual Inductance (conclusions)



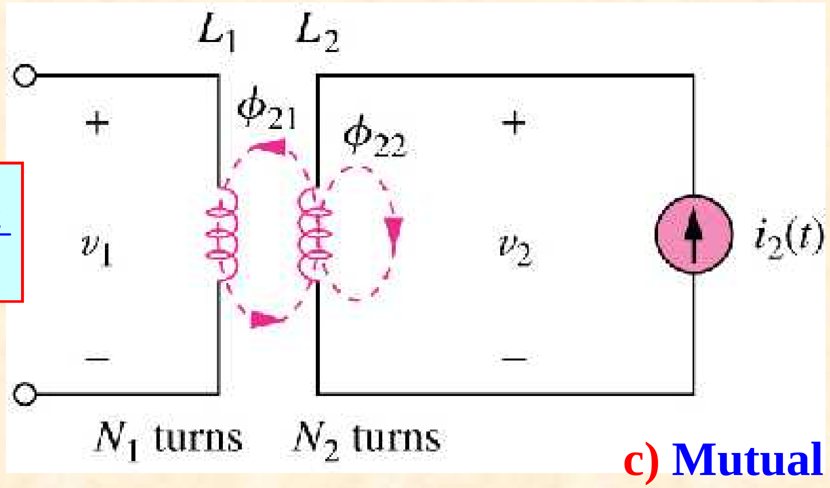
a) Magnetic flux produced by a single coil



$$v_2 = M_{21}\frac{di_1}{dt}$$

b) Mutual inductance M_{21} of coil 2 with respect to coil 1

$$v_1 = M_{12}\frac{di_2}{dt}$$



c) Mutual inductance of M_{12} of coil 1 with respect to coil 2

Terms & Definitions

- ✓ **Inductor-** A device that introduces inductance into an electrical circuit (usually a coil)
- ✓ **Inductance-** The property of an electric circuit when a varying current induces an EMF in that circuit or another circuit
- ✓ **Self-inductance-** The property of an electric circuit when an EMF is induced in that circuit by a change of current
- ✓ **Henry -** The unit of inductance
- ✓ **Permeability-** The measure of the ease with which material will pass lines of flux
- ✓ **Mutual Inductance-** The property of two circuits whereby an EMF is induced in one circuit by a change of current in the other

Mutual inductance in terms of self inductances

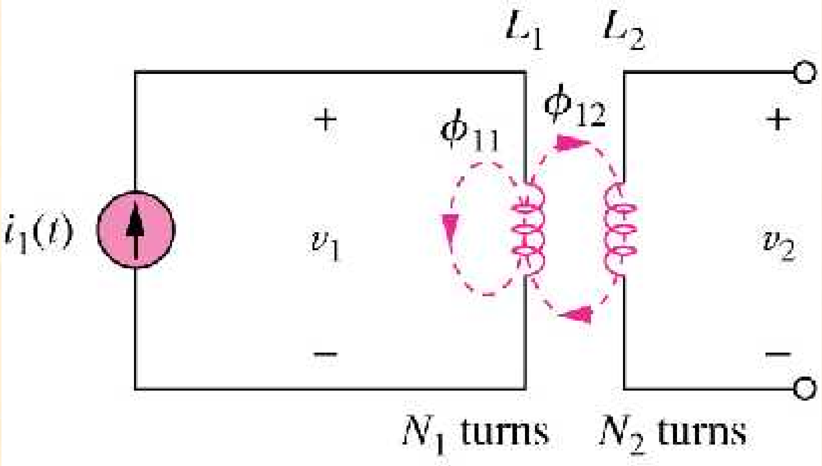
$$L_1 = N_1^2 \, P_1$$

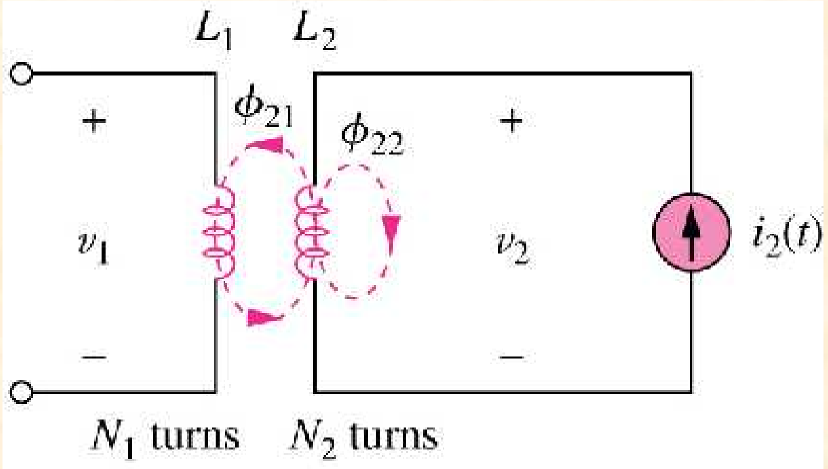
$$L_2 = N_2^2 \, P_2$$

$$L_1 L_2 = N_1^2 N_2^2 \, P_1 P_2$$

$$P_1 = P_{11} + P_{21}$$

$$P_2 = P_{22} + P_{12}$$





Mutual inductance in terms of self inductances

$$L_1 L_2 = N_1^2 N_2^2 \left(P_{11} + P_{21}\right)\left(P_{22} + P_{12}\right)$$

For a linear system, $P_{12} = P_{21}$

$$L_1 L_2 = N_1^2 N_2^2 \, P_{12}^2 \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$

Mutual inductance in terms of self inductances

$$L_1L_2 = (N_1N_2\,P_{12})^2\left(1+\frac{P_{11}}{P_{12}}\right)\left(1+\frac{P_{22}}{P_{12}}\right)$$

$$L_1L_2 = M^2\left(1+\frac{P_{11}}{P_{12}}\right)\left(1+\frac{P_{22}}{P_{12}}\right)$$

$$\frac{1}{k^2} = \left(1+\frac{P_{11}}{P_{12}}\right)\left(1+\frac{P_{22}}{P_{12}}\right)$$

$$M^2 = k^2 L_1 L_2$$

Mutual inductance in terms of self inductances

➢ The mutual inductance can be written in terms of self inductances as:

$$\boxed{M = k\sqrt{L_1 L_2}}$$

✓ The constant **"k"** is called the **coupling coefficient**

$$\frac{1}{k^2} = \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$ ⇨ **Must be greater than 1**

✓ Therefore k ⇨ **Must be less than 1**

Coupling Coefficient

- The **coupling coefficient** "**k**" is a measure of the percentage of flux from one coil that links another coil (a measure of the magnetic coupling between two coils). The coupling coefficient for 2 mutual inductors is given by:

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

- The **coupling coefficient** "**k**" depends on the closeness of two coils, their core, their orientation and their winding

Coupling Coefficient

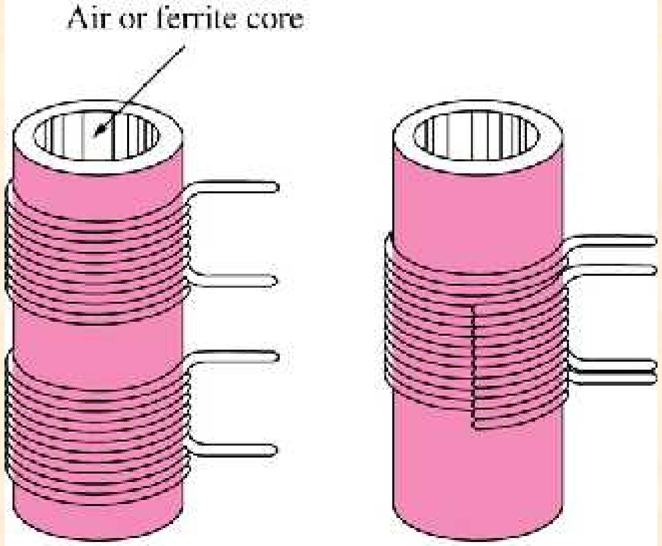
➢ If $k > 0.5$, then most of the flux from the one coil links the other and the coils are said to be **tightly coupled**

➢ If $k < 0.5$, then most of the flux is not shared between the 2 coils and in this case the coils are said to be **loosely coupled**

❑ **Range of k : $0 \le k \le 1$**

✓ $k = 0$ means the two coils are **not coupled**

✓ $k = 1$ means the two coils are **perfectly coupled**



Loosely coupled coil **Tightly coupled coil**

Coupling Coefficient

k can be expressed in terms of flux as

$$k = \frac{\phi_{12}}{\phi_{11} + \phi_{12}}$$

or $$k = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

$k = 1$ means perfect coupling.

$$\Rightarrow \phi_{11} = \phi_{22} = 0$$